STRUCTURE OF A STRONG SHOCK WAVE IN A PLASMA

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According to existing concepts as a result of high electronic thermal conductivity an extended region of hot electrons is formed in the front of a rather strong shock wave propagating in a plasma and an abrupt (isothermal with respect to the electron temperature) density jump, due to the ion viscosity, is introduced into that region [1, 2]. The isothermal jump is a necessary consequence of the high electronic thermal conductivity [1, 3]. As the Mach number increases, however, the thickness of the density jump decreases while the mean free path of the particles increases and when these quantities become equal, the hydrodynamic approximation becomes unacceptable, generally speaking.

Moreover, with increasing Mach number M the energy of the directed motion grows in comparison with the thermal energy of the plasma. When this inequality becomes strong, virtually the entire energy of the directed motion of the ions will be expended on heating the electrons [4]. Since the ions almost do not heat up in this process, it thus follows that the density jump in the shock-wave front should be due to not to the ionic viscosity but to other effects. On the basis of these concepts, we propose a shock-wave structure in which the balance of the material, momentum, and energy fluxes is ensured by the formation, in the wave front, of a collisionless potential jump corresponding to the ionic-acoustic shock wave [5-7] and dissipation of the flux of ions reflected from that potential jump.

On the basis of general concepts let us analyze the structure of strong shock waves in a plasma without a magnetic field. As is known [1], jumps in the parameters in a shock-wave front in a plasma as in an ordinary gas satisfy the relations

$$\begin{gathered} n_0 u = n_2 u_2,\ p_0 + n_0 m u^2 = p_2 + n_2 m u_2^2, \\ w_0 + u^2/2 = w_2 + u_2^2/2, \end{gathered} \tag{1}$$

where the subscript 0 pertains to parameters of the unperturbed plasma and the subscript 2 pertains to parameters behind the shock-wave front, $p = n_i T_i + n_e T_e$ (n_i, n_e are the ion and electron densities), $w = (\gamma/(\gamma-1))(T_i + T_e)$ is the enthalpy, u is the velocity of the shock wave, and γ is the adiabatic exponent.

The relations taking into account the conservation of the fluxes of matter, momentum, and energy in a collisionless ionic-acoustic jump with allowance for the jump in the electric potential generated by reflected ions have the form [7]

$$\begin{gathered} (n_f - n_r)u_1 = n_2 u_2,\ n_0 u_s^2 \exp\psi_1 + (n_f + n_r)u_1^2 = \\ = n_2 u_2^2 + n_0 u_s^2 \exp\psi, \\ (1/2)u_1^2 + \psi_1 u_s^2 = (1/2)u_2^2 + \psi u_s^2,\ n_f + n_r = \\ = n_0 \exp\psi_1,\ n_2 = n_0 \exp\psi. \end{gathered} \tag{2}$$

Here n_0 is the concentration of the unperturbed plasma, u_1 is the velocity of the matter (in the system of the wave) directly in front of the ionic-acoustic jump, n_2 and u_2 are the plasma density and velocity behind the shock-wave front, n_r is the density of reflected ions, n_f is the density of ions running into the ionic-acoustic jump, $\Psi_1 = e\varphi_1/T_e$ is the potential produced by the reflected ions, and $\psi = e\varphi/T_e$ is the total potential jump in the shock wave.

Equations (2) are valid, generally speaking, for specific values of M at which the potential distribution in the wave front is monotonic. At other values of M the potential distribution becomes oscillatory [6, 7] and Eqs. (2) give a fairly accurate estimate of the average values of the parameters behind the ionic-acoustic jump.

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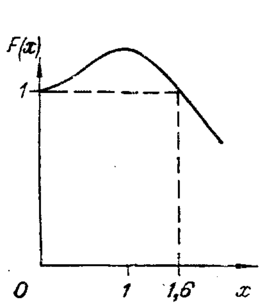

Fig. 1

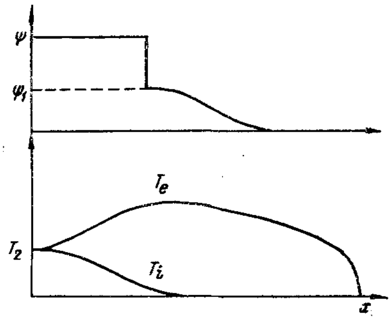

Fig. 2

From (2) we obtain the following expression for the Mach number M_1 at the ionic-acoustic jump:

$$(1+M_1^2)\exp\left[\frac{1}{2}(M_2^2-M_1^2)\right]-M_2^2-1=0 \qquad (3)$$
$$(M_1=u_1/u_s,\ M_2=u_2/u_s,\ u_s=\sqrt{T_e/m}).$$

The solution of Eq. (3) reduces to a search for those values of the function $F(x) = (1+x^2)\exp[-(1/2)x^2]$, to which two different values of x correspond. The graph of the function F(x) is shown in Fig. 1. Clearly, $x = M_1$ at $x > 1$ and $x = M_2$ at $x < 1$. At $F(x) = 1$ we have the limiting value $M_1 \approx 1.6$, which accords with the result of [5].

We must ascertain how the general relations (1) can be satisfied with allowance for the fact that a potential jump corresponding to the collisionless ionic-acoustic shock wave described by Eqs. (2) is formed in the shock-wave front. Since the plasma temperature does not change at the potential jump in the ionic-acoustic shock wave, the heating indicated by the general relations should occur during dissipation of the flux of ions reflected from the potential jump. The reflected ions relax mainly on electrons [4] and the distance over which the reflected ions relax is substantially shorter than the size of the hot electron zone. The corresponding density (pedestal) can be assumed to be isothermal and electrons here are described by the Boltzmann distribution while the width of the jump is determined by the mean free path of the reflected ions.

Obviously, the general relations (1) should be satisfied at the complete density jump, taking into account the jump upon dissipation of the flux of reflected ions and the collisionless jump. Since the ions remain practically cold, in order to ensure the necessary pressure jump the electron temperature directly behind the density jump should be twice as high as follows from the general relations (1) on the assumption that the plasma is isothermal behind the front. Only in this case will the general relations be satisfied as a result of the equalization of the temperatures. We have in mind here the dissipation of the reflected ion flux in Coulomb collisions, but the qualitative picture is preserved for dissipation in collective effects as well.

The equations of conservation of the flux of matter, momentum, and energy on the pedestal on the assumption of an isothermal density jump have the form

$$n_0u=(n_f-n_r)u_1,\ n_0u^2=n_0u_s^2\exp\psi_1+$$
$$+(n_f+n_r)u_1^2,\ (1/2)u^2=(1/2)u_1^2+\psi_1u_s^2+u_s^2. \qquad (4)$$

These equations, along with (2), form a closed system which determines all the quantities in the front in terms of the parameters of the unperturbed plasma and the shock-wave velocity.

The structure formed thus is self-consistent in the sense that the reflected ions produce the conditions for the formation of a collisionless shock wave, which in turn ensures the necessary flux of reflected ions.

We require that the conditions behind the front of a collisionless ionic-acoustic shock wave accord with the conditions behind the front of the entire shock wave described by Eqs. (1) (except for the condition of isothermality). Then, using the first of Eqs. (1), we can rewrite Eq. (3) as:

$$(1+M_1^2)\exp\left[\frac{1}{2}\left(\frac{n_0^2}{n_2^2}M^{*2}-M_1^2\right)\right]-\frac{n_0^2}{n_2^2}M^{*2}-1=0 \quad (M^*=u/u_s). \tag{5}$$

It is significant here that M^* tends to an asymptotic value when the velocity of the shock wave increases without bound. We assume that the shock wave propagates in an isothermal plasma, i.e., $T_{io} = T_{eo}$. Since a hot electron zone exists in the wave front, the density jump moves with an essentially nonisothermal plasma. On the other hand, we know that the temperature jump in the front of a strong shock wave is proportional to M^2, where $M = u/\sqrt{\gamma(T_e+T_i)/m}$. Consequently, M^* determined for the density jump from the temperature of hot electrons differs substantially from the M determined from the temperature of the unperturbed plasma and tends to a constant value as M increases without bound. Since the electron temperature in the region of the density jump is twice as high as the temperature behind the shock wave, then $M^* = M\sqrt{\gamma T_0/T_2}$. From Eqs. (1) in the case of a strong shock wave it follows that the asymptotic value is $M^* = (\gamma+1)/\sqrt{2(\gamma-1)}$. At $\gamma = 5/3$ we have $M^* = 2.3$.

From Eq. (5) at $M^* = 2.3$, i.e., as $M \to \infty$ and $n_2/n_0 \to 4$, we obtain $M_1 = 1.37$. From Eqs. (2) for $M_1 = 1.37$ we find $\psi_1 = 0.6$, $n_r/n_0 = 0.08$, $n_1/n_0 = 1.8$, and $\psi = 1.4$. As long as the shock wave is sufficiently strong, M^* and n_2/n_0 change only slightly as M increases. The value of M_1 remains virtually constant in this case. For example, at M = 10 we have $M^* = 2.27$ and $n_2/n_0 = 3.88$ and the corresponding values are $M_1 = 1.36$ and $n_r/n_0 = 0.07$.

As follows from the calculations in [6], M = 10 ($M_1 = 1.36$, $T_e/T_i \sim 60$) corresponds roughly to the minimum M at which a collisionless potential jump can form (the potential has monotonic profile). At lower values of M the energy of the reflected ion flux is insufficient to ensure that the plasma is nonisothermal as required.

The structure of the potential in the shock-wave front and the distribution of the electron and ion temperatures are shown in Fig. 2. A potential jump with respect to ψ_1 is formed in the relaxation region of the flux of reflected ions. The jump from ψ_1 to ψ is a collisionless shock wave. The distribution of the electron temperature has a maximum in the region of the ionic-acoustic jump, since the flux of reflected ions dissipates ahead of it. The electron temperature then decreases smoothly and the ion temperature rises smoothly to the value behind the shock-wave front, which follows from the general relations at a shock discontinuity (1).

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